CHIRAL SYMMETRY AND THE PION MASS*

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The realization of chiral SU(2)⊗SU(2) symmetry by nonlinear field transformations is considered from the standpoint of axiomatic field theory. We show that the communication relations of chiral symmetry are incompatible with the assumption that the pion has nonzero mass.

The success of the Adler-Weisberger sum rule[1] has stimulated interest in "chiral symmetry,"[2,3] in which the integrals of the time components of the hadronic weak current are assumed to (exist and) generate the Lie algebra of SU(2) ⊗SU(2). These integrals can be constants of the motion only if the corresponding currents have vanishing four-divergence. However, simple invariance arguments[4] show that the observed decay of the charged pion requires that the divergence of the axial-vector current have at least one matrix element proportional to m_π^2. Weinberg[3] has discussed the question of formulating chiral symmetry in terms of a nonlinear transformation law involving only the pion field, and has found that the commutation relations

$$[T_i, \pi_j(x)] = i\epsilon_{ijk}\pi_k(x),$$
$$i[X_i, \pi_j(x)] = f_{ij}[\pi_1(x), \pi_2(x), \pi_3(x)], \tag{1}$$

do indeed have solutions compatible with the commutation rules of SU(2)⊗SU(2):

$$[T_i, T_j] = i\epsilon_{ijk}T_k,$$
$$[T_i, X_j] = i\epsilon_{ijk}X_k,$$
$$[X_i, X_j] = i\epsilon_{ijk}T_k. \tag{2}$$

He has then discussed the problem of constructing a Lagrangian density which is invariant under infinitesimal transformations by the X's and T's, and has shown that such a Lagrangian cannot contain (nonconstant) terms of the form $m_\pi^2 f(\pi^2)$. Thus, independently of arguments about the weak interactions, he shows that exact chiral symmetry [in the sense of Eq. (1)] is incompatible with nonzero pion mass.

The proposed justification for selecting one of the many possible transformation laws for π is the Haag-Nishijima-Zimmerman theorem,[5] which states that the S matrix is invariant under a large class of redefinitions of the fields. Since this is valid without reference to a Lagrangian formulation, we have considered the problem of nonlinear realizations in the context of axiomatic field theory. We shall show that the assumption of nonzero pion mass is still incompatible with properties (1) and (2). We make use of Weinberg's important remark that the commutation relations can always[3,6] be brought to the form

$$i[X_i, \pi_j(x)] = \delta_{ij} + \pi_i(x)\pi_j(x) \tag{3}$$

by suitable definition of the pion field. If we now assume the usual axioms of field theory[7] together with the spectral condition, we obtain a contradiction, as follows. Let $U(\Lambda, a)$ be the representation operator of an element of the (proper, orthochronous) Poincaré group. Transforming Eq. (3), we find

$$i[UX_iU^{-1}, \pi_j(x')] = \delta_{ij} + \pi_i(x')\pi_j(x'), \tag{4}$$

$$x' = \Lambda^{-1}(x-a). \tag{5}$$

But (3) is assumed to hold for all x; hence

$$[UX_iU^{-1} - X_i, \pi_j(x)] = 0. \tag{6}$$

(When chiral symmetry is extended to include fields other than the pion,[3] a similar argument shows that $UX_iU^{-1} - X_i$ commutes with all the fields in the theory.) A theorem due to Ruelle[8] suggests that this difference must then be a multiple of the identity:

$$U(\Lambda, a)X_iU(\Lambda, a)^{-1} - X_i = c_i(\Lambda, a). \tag{7}$$

If we apply a second Poincaré transformation, we see that the $c_i(\Lambda, a)$ form an additive representation of the Poincaré group. Hence, since the Poincaré group has no nontrivial Abelian quotient groups,[9] we must have $c_i(\Lambda, a) = 0$. In particular,

$$[P_\mu, X_i] = 0. \tag{8}$$

Then X_i takes the vacuum into a state with four-momentum zero, which, if there are no mass-zero particles, can only be a multiple of the vacuum. From either the commutation relations [Eq. (2)] or the odd parity of X, we see that the multiple must be zero; i.e., X_i annihilates the vacuum.

We now obtain a contradiction by taking the vacuum expectation value of Eq. (3)[10]:

$$\langle 0|(X_3\pi_3-\pi_3X_3)|0\rangle = 0-0$$
$$=\langle 0|(1+\pi_3\pi_3)|0\rangle$$
$$=1+\sum_n|\langle 0|\pi_3|n\rangle|^2>0.$$

Reviewing the arguments which led to this contradiction, we see that any attempt to formulate a relativistic quantum field theory with chiral transformations given by Eq. (1) must encounter difficulty. In particular, the "usual mass-spectrum condition" cannot be imposed—that is, the pion must have zero mass.[11]

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[5]R. Haag, Phys. Rev. 112, 669 (1958); K. Nishijima, Phys. Rev. 111, 995 (1958); M. Zimmerman, Nuovo Cimento 10, 597 (1958).

[6]This is not precisely true, since it can be shown that there are two distinguishable classes of nonlinear realizations. This point, which we shall discuss elsewhere, does not alter our conclusions.

[7]R. F. Streater and A. S. Wightman, PCT, Spin and Statistics, and All That (W. A. Benjamin, Inc., New York, 1964). Note that by interpreting the fields $\pi_i(x)$ as operators, we are already going far beyond their "phenomenological interpretation" as a collection of greek and latin letters.

[8]D. Ruelle, Helv. Phys. Acta 35, 147 (1962), Appendix. The proof applies only to bounded operators. Our argument can be recast in a form involving the chiral group operators $e^{ia\cdot X}$ rather than the generators; Ruelle's theorem then applies rigorously, and we are led to the same final result.

[9]We can see directly that if $UXU^{-1}X$ is c-number it must vanish, by taking its vacuum expectation value and noting that $U|0\rangle=|0\rangle$. We thank Professor K. Kowalski for pointing out this simpler proof.

[10]A smearing function f should be applied in order to make the left-hand side nonsingular. $\pi^f|0\rangle$ is normalizable and is assumed to belong to the domain of X. If f is chosen to be non-negative, the right-hand side remains positive definite.

[11]This does not answer the converse question of whether permitting the pion to have zero mass does make chiral symmetry possible. [Some complications associated with zero mass are discussed by J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962), especially Sec. III.] We regard massless fields as terra incognita and dare not guess the detailed geography.

GAUGE INVARIANCE, BROKEN SYMMETRIES, AND FIELD-CURRENT IDENTITIES

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It is shown that (approximate) field-current identities appear in a natural way in a Yang-Mills–type theory if the local symmetry is spontaneously broken.

It has been suggested[1,2] that the idea of vector dominance in particle interactions can be formulated in terms of (approximate) identities between the vector currents taking part in the interactions and vector fields. An attempt at a concrete realization of these identities in a consistent Lagrangian formalism leads one to a Lagrangian which is similar[2] to the one in Yang-Mills–type theories[3] except for the mass term of the vector particles. Local gauge-invariance is, therefore, lost and does not play any role in the theory. On the other hand, it has been shown[4,5] that it is possible to obtain massive vector mesons in a Yang-Mills–type theory if the local