Skronn, H. Wahl, and K. Wegener, Phys. Letters 29B, 41 (1969), and at CEA up to $-t = 1.2$ (GeV/c)^2 by Z. Bar-Yam, J. de Pagter, J. Dowd, W. Kern, P. D. Luckey, and L. S. Osborne. Preliminary results of the CEA group and additional preliminary DESY data were included in the report by K. Lübelsmeyer, Ref. 3.

[5]π^- production with polarized photons was measured at DESY by C. Geweniger, P. Heide, U. Kötz, R. A. Lewis, P. Schmüser, H. J. Skronn, H. Wahl, and K. Wegener, Phys. Letters 28B, 155 (1968), and at CEA by Z. Bar-Yam, J. de Pagter, J. Dowd, and W. Kern, *cf.* the report by B. Richter in the *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968,* edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), and Bull. Am. Phys. Soc. 14, 542 (1969). Recent preliminary results were included in the report by K. Lübelsmeyer, Ref. 3.

[6]R. Diebold, Phys. Rev. Letters 22, 204 (1969), and Stanford Linear Accelerator Center Report No. SLAC-PUB-673, 1969 (unpublished).

[7]H. Überall, Phys. Rev. 103, 1055 (1956); G. Barbiellini, G. Bologna, G. Diambrini, and G. P. Murtas, Phys. Rev. Letters 8, 454 (1962); G. Diambrini Palazzi, Rev. Mod. Phys. 40, 611 (1968).

[8]A description of the precision goniometer for the remotely controlled alignment of the diamond is contained in IEEE Trans. Nucl. Sci. 16, No. 1, 648 (1969).

[9]Z. Bar-Yam, V. Elings, D. Garelick, R. Lewis, W. Lobar, P. D. Luckey, L. Osborne, S. Tazzari, J. Uglum, and R. Fessel, Nucl. Instr. Methods 56, 1 (1967).

[10]Preliminary results of this experiment, as reported in Abstract No. 51 of the *Proceedings of the Fourth International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969,* edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970), and cited in the report by K. Lübelsmeyer, *ibid.*, p. 45, included a point at $-t = 2.4$ (GeV/c)^2 which showed A^- to be -0.86 ± 0.21. Confirmation of this point and additional measurements in its vicinity are needed and are being pursued by our group.

[11]See, for instance, M. Krammer and D. Schildknecht, Nucl. Phys. B7, 583 (1968).

[12]R. Diebold and J. A. Poirier, Phys. Rev. Letters 22, 255, 692(E), 906 (1969).

[13]Z. G. T. Guiragossián and A. Levy, Phys. Letters 30B, 48 (1969).

[14]See, for instance, D. Schildknecht, DESY Report No. 69/41 (unpublished).

POSSIBLE DIFFICULTY WITH THE "EIKONAL PICTURE" OF HIGH-ENERGY INTERACTIONS*

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We present a heuristic argument suggesting that at high energies the effect of multiple scattering off the energy shell becomes important for an understanding of the propagation of one elementary particle through another.

The physical picture of high-energy scattering at small angles as a diffractive effect[1] plays a large role in our present understanding of these processes. Although mathematically this picture has been related to the eikonal approximation in potential scattering, there are some physical differences, having to do with the "adiabatic" or "closure" concept, which may become important at high energy. In particular, when we consider the propagation of strongly interacting particles through a dense medium, the following problem arises. If we know that only interactions occurring along the classical trajectory are important we can understand that on-shell scattering is more important than off-shell. If, on the other hand, we know that on-shell scattering dominates we can understand (by a stationary-phase argument) that interactions occurring along the classical trajectory are the most important ones.[2] Of course these two statements together do not imply that either of the hypotheses (dominance of the classical trajectory or dominance of on-shell effects) is true at high energies. A very simple picture, described below, suggests that in the "closure approximation" they are both false.

The picture we use is a descendant of the algebraic formulation of multiple scattering introduced by Foldy.[3] We isolate three points in the medium, labeled 1, 2, and 3 in Fig. 1, and ask for the coherent wave incident on point 3, due to scatterings at 1 and 2, averaged over all positions of point 2. This averaging represents the "closure approximation." We consider first the wave incident on 2, due to an interaction at point 1. Now, because point 2 may come very close to point 1, we should use not only the outgoing part of the scattered wave, but also the standing wave, or "near zone" parts, which represent the effects of (half) off-energy-shell scattering. We represent this for an incident plane wave of momentum k by the expression

$$\psi(r,\theta)-\psi_{\rm in}(r,\theta)=f(\theta,k^2)e^{ikr}/r+g(\theta,k^2,r)e^{-\mu r}/r.$$

Since the wave function must be regular at the point 1 ($r=0$), we have the limit

$$g(\theta,k^2,r)\underset{r\to 0}{\longrightarrow}-f(\theta,k^2),\quad \psi-\psi_{\rm in}=f(\theta,k^2)\left(\frac{e^{ikr}}{r}-\frac{e^{-\mu r}}{r}\right)+\text{rest.}$$

In the on-shell approximation only the term e^{ikr}/r is retained. The coherent amplitude arriving at point 3 is given in the algebraic approximation by

$$\int d^3 2\,\psi_{1\to2\to3}=\int d^3 2\frac{1}{r}\frac{1}{r'}f(\theta,k^2)f(\theta',k^2)(e^{ikr}-e^{-\mu r})(e^{ikr'}-e^{-\mu r'})+\text{various other terms.}$$

To illustrate our point we compare the "pure scattered wave" contribution with the "pure standing wave" contribution:

$$\begin{Bmatrix}\psi_{sc}\\ \psi_{st}\end{Bmatrix}\equiv\int d^3 2\frac{1}{r}\frac{1}{r'}f(\theta,k^2)f(\theta',k^2)\begin{Bmatrix}e^{ik|r+r'|}\\ e^{-\mu|r+r'|}\end{Bmatrix}.$$

We examine the integral in ellipsoidal coordinates,

$$\eta\equiv r+r',\quad \xi\equiv r-r',\quad \varphi=\text{azimuthal angle:}$$

$$\begin{Bmatrix}\psi_{sc}\\ \psi_{st}\end{Bmatrix}=\int d\varphi\int d\xi\int d\eta\left|\frac{\partial(x_1x_2x_3)}{\partial(\eta,\xi,\varphi)}\right|\frac{1}{rr'}f(\theta,k^2)f(\theta',k^2)\begin{Bmatrix}e^{ik\eta}\\ e^{-\mu\eta}\end{Bmatrix}.$$

The portion called ψ_{st} has some k dependence due to the factors $f(\theta,k^2)$ and $f(\theta',k^2)$. The portion called ψ_{sc} has an additional k dependence since, by the Riemann-Lebesgue lemma, it is less than the variation of all the other factors divided by k. As k becomes infinite we must find

$$\psi_{sc}/\psi_{st}\to 0.$$

Thus, rather than justifying the two hypotheses mentioned earlier, we find that both become somewhat doubtful. The crucial points in this argument have been the following: (1) We know that, because of the regularity of the wave function, there must be a standing-wave part which we have represented algebraically by $e^{-\mu r}/r$. (2) We integrate coherently over all positions of the second point (the adiabatic assumption). (3) We do not exclude any region of integration. The significance of the third point is the following. In scattering from a gas or a lattice the particles 1 and 2, or 2 and 3, cannot come very close to each other. The situation is then controlled (at all reasonable energies) by the exponential (short-range) character of the standing-wave parts. In nuclei, while the situation is not quite so clear cut, there are hard cores which hold the particles apart.

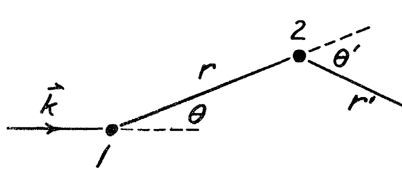

Fig. 1. Points 1, 2, and 3 are typical points in an extended medium. We approximate the wave incident on 3 due to an intermediate scattering at 2 by the product (the "algebraic approximation") of the wave at 2 due to interaction at 1 and the wave at 3 due to interaction at 2.

Recently, however, there has been a great deal of discussion of interactions of elementary particles using techniques related to this general viewpoint. In this case the "constituents" are at present entirely conceptual and we have no reason to restrict the region of integration. Thus a correct analysis of pictures such as the "parton" model[4] might require more careful analysis of off-shell effects typified here by the standing part of the scattered wave. Finally, there has been considerable analysis of the eikonal approximation in relativistic perturbation theory. Insofar as Feynman diagrams reflect the naive space-time pictures employed here, it should be possible to find these off-shell effects in the high-energy limit of perturbation theory.

The detailed analysis of Feynman diagrams by Cheng and Wu[5] reveals that in individual diagrams off-shell effects are important, but in gauge-invariant sums they cancel. One must of course wonder what happens in a theory with no need for gauge invariance. A related idea appears then in the work of Lee,[6] who has studied an operator formulation[7] of the droplet model in which the eikonal is replaced by an operator source function. He finds that this source function must be the density of a conserved charge in order to have finite differential cross section at infinite energies.

This remark originated in an unpublished investigation of the propagation of neutral K mesons through matter. It is a pleasure to acknowledge several discussions with Professor L. Foldy and Professor C. N. Yang.

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[1]R. Serber, Phys. Rev. Letters 10, 357 (1964); R. Glauber, in *Lectures in Theoretical Physics: High Energy Collision Theory,* edited by W. E. Britten and L. G. Dunham (Interscience, New York, 1959), Vol. 1.

[2]Throughout this paper we shall abhor rigor. Actually there has been a great deal of success in establishing these points more rigorously.

[3]L. L. Foldy, Phys. Rev. 67, 107 (1945).

[4]R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[5]T. T. Wu, private communication. The work of these authors may be traced from a recent paper, H. Cheng and T. T. Wu, Phys. Rev. Letters 24, 759 (1970).

[6]B. W. Lee, Phys. Rev. D (to be published).

[7]T. T. Chou and C. N. Yang, Phys. Rev. 175, 1832 (1968), and papers cited therein.

OBSERVATION OF A FOUR-PION RESONANCE AT 1630 MeV in K^+p INTERACTIONS AT 10 GeV/c†

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We report the observation of a new four-pion resonance $\rho(1630)$ with mass 1632 ± 23 MeV and width 110^{+40}_{-30} MeV produced in the channel $K^+p \to K^0\pi^+\pi^+\pi^-\pi^0p$ and decaying predominantly to $\rho^{0,-}\pi^{0,+}\pi^+$ and $\omega\pi^+$. No decay is observed into $\pi^+\pi^0$, $K^+\bar{K}^0$, $\rho^+\rho^0$, or $A_2\pi$, so that association with the g is precluded and the spin and parity probably lie in the unnatural series. The $\rho(1630)$ is associated with a $K\rho(1630)$ enhancement near threshold. This and other production characteristics are consistent with a double Regge pole exchange mechanism.

The four-pion decay mode of a boson resonance in the 1700-MeV region was first observed by Bellini et al.[1] Confirmation has come from several other πp production experiments[2-7] and $\bar{p}p$ annihilation experiments.[8,9] Evidence for decays via $\omega\pi$,[3,5,7] $A_2\pi$,[2,4,8] and $\rho\rho$[3-5] has also been claimed. The evidence points to the existence of a resonance[10] $\rho(1710)$ decaying predominantly to four pions but in individual experiments[5] this mode has been difficult to disentangle from possible four-pion decay modes of the $g(1660)$. The CERN Missing Mass Spectrometer Group (MMS)[11] observe several enhancements in this mass region which are appreciably narrower than those claimed in bubble chamber experiments. We report here the observation of a new resonance $\rho(1630)$ produced in the reaction

$$K^+p \to K^0\rho(1630)^+p \tag{1}$$

at 10 GeV/c incident momentum and decaying predominantly to $\rho^{0,-}\pi^{0,+}\pi^+$ and $\omega\pi^+$, but with no detectable $\pi^0\pi^+$, $\rho^0\rho^+$, $A_2^{0,+}\pi^{+,0}$, or $\bar{K}^0K^+$ decay modes.

Experimental details. – A double scan was made of 500 000 pictures of the 2-m CERN hydrogen bubble chamber exposed to a 10-GeV/c separated K^+-meson beam for the topologies two-prong + V^0 and four-prong + V^0. After measurement, kinematic analysis and ionization checks, the yields of uniquely identified events are as follows[12]:

$$K^+p \to K^0\pi^+\pi^+\pi^-\pi^0p, \quad 2984 \text{ events}; \tag{2}$$

$$K^+p \to K^0\pi^+\pi^0p, \quad 4463 \text{ events}; \tag{3}$$